Pairwise Dissimilarity and Risk-Seeking Portfolio Construction

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Abstract

I propose a heuristic portfolio construction rule that weights assets according to their dissimilarity from the remaining holdings. The methodology is insensitive to estimation error and does not assume a return-generating process. Relative to the 1/N rule, in a sector rotation setting, the dissimilarity approach delivers annual excess returns of 6 to 121 bps while maintaining comparable levels of diversification, volatility, tail risk, and turnover. This outperformance reflects compensation for bearing tail risk and harnessing momentum by dynamically increasing the weights of rapidly appreciating assets.

Keywords: Portfolio Construction, Pairwise Dissimilarity, Tail Risk.

JEL Classification: C58, G11.

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1 Introduction

Investors often seek risk in a long-only setting. For example, to maximize lifetime wealth, individuals with long investment horizons should prioritize returns over the risk-return tradeoff and allocate to risky assets early in their careers. In some cases, maximizing next-period returns may be necessary, even at the expense of future gains or aggregate wealth accumulation. To that end, when facing shortfalls, pension plans tend to increase holdings of risky assets with uncertain but high expected returns (Giesecke and Rauh (2023)). Existing portfolio construction methods do not adequately address this problem; for instance, the maximum return portfolio on the mean-variance efficient frontier consists of a single asset. In this paper, I propose a heuristic methodology based on pairwise asset dissimilarity that results in portfolios with higher returns and higher risk while maintaining diversification, volatility, tail risk, and turnover at levels comparable to the baseline 1/N rule.

I propose weighting assets in proportion to their dissimilarity from the rest of the portfolio. For $N > 1$, where N is the number of assets, i and j index the assets, and $D(i, j)$ denotes the pairwise dissimilarity between assets i and j, the weight of asset i is given by:

$$w_i = 0.5 \cdot \frac{\sum_{j=1}^{N} D(i, j)}{\sum_{i=1}^{N} \sum_{j=1}^{N} D(i, j)}.$$

From the definition alone, it is clear that this methodology preserves important features of the 1/N rule. In realistic settings, the weighting scheme inherently avoids overconcentration; exposure to common factors ensures that every asset receives some weight. This heuristic is also transparent and easy to implement. It is rules-based and long-only, relies solely on past returns in a manner that scales to large asset universes, and does not require explicit estimation of the covariance matrix or expected future returns. As a result, it is particularly useful in applications where the number of assets exceeds the length of the return history. Moreover, the simplicity of the weighting methodology is appealing from a practitioner's perspective. Heuristic portfolio construction methods based on simple financial metrics (e.g., capitalization-weighted portfolios) are already the most widely used weighting schemes by

AUM. Quantitative heuristics show promising results for enhancing portfolio characteristics and performance; for instance, Chow et al. (2014) highlight the advantages of inverse volatility portfolios.

For transparency, replicability, and computational tractability, I measure pairwise dissimilarity, $D(i, j)$, using the two-sample Kolmogorov-Smirnov (KS) and Anderson-Darling (AD) test statistics, which offer three significant advantages. First, both the KS and AD statistics capture the entire return distribution rather than a limited number of moments. Second, these measures are nonparametric and therefore generally robust to measurement error. Third, pairwise asset dissimilarity measured using the KS and AD statistics does not explicitly or implicitly rely on any assumed return-generating process or factor structure.

KS and AD statistics produce portfolios with higher risk and return relative to the 1/N benchmark by overweighting assets with elevated idiosyncratic and systematic risks. However, the general approach is flexible. It is possible to define a custom dissimilarity function that targets a specific objective or incorporates subjective beliefs. After all, most of the properties of resulting portfolios are inherited directly from the dissimilarity function rather than the weighting scheme. Such flexibility makes the approach appealing for commercial applications, which demand tailored solutions based on proprietary research or information.

I test the proposed methodology by implementing a sector rotation strategy using 12 industry portfolios. Industries earn different risk premia and perform differently across economic and market conditions, so rotating exposure across sectors would result in a meanigful deviation from the baseline 1/N rule. Moreover, as shown in Candes et al. (2025), assets grouped around a common theme (measured by pairwise correlations) "have risk levels above predictions," suggesting that the out-of-sample sector rotation setting is a useful gauge of risk management properties. From a practitioner's perspective, sector rotation is a popular actively managed investment strategy, making a quantitative, rules-based alternative a cost-effective option and a useful benchmark.

Out-of-sample, in the sector rotation setting, the average annual excess performance relative to the 1/N benchmark ranges from 6 to 121 basis points. The outperformance is

achieved without compromising diversification, volatility, tail risk, or turnover characteristics. Moreover, the dissimilarity-based portfolios have appealing implementation properties: they are robust to the choice of estimation window length and are insensitive to estimation error.

To further demonstrate the applicability of the pairwise dissimilarity heuristic, I apply it to asset allocation and individual equity portfolio construction. In the asset allocation setting, the average annual excess performance relative to the 1/N benchmark is approximately 25 basis points. In the individual equity portfolio setting, maximum outperformance reaches 579 basis points. Such extreme outperformance, however, relies on an asset universe in which each asset has limited downside risk while one (or a few) assets experience rapid price appreciation. In this case, the dissimilarity of the appreciating asset relative to the rest of the universe increases, leading to higher portfolio weight, in turn capturing a momentum effect. This is a key feature of the heuristic: when assets are carefully selected to limit downside, rapid appreciation of any single asset is more likely than depreciation. Consequently, the portfolio is more likely to capture the upside rather than downside, leading to long-run outperformance. Absent rapid price changes, portfolio performance remains roughly in line with the market.

The outperformance of dissimilarity strategies is also related to the documented compensation for bearing tail risks. Empirical dissimilarity (measured by the KS and AD statistics) and the corresponding asset weights increase with the idiosyncratic risk, with the AD statistic being particularly sensitive to the tails of the return distribution. High idiosyncratic risk assets often display skewness and heavy-tailed behavior, which are priced across various asset classes (Harvey and Siddique (2000), Kelly and Jiang (2014), Hua et al. (2021), Schreindorfer and Sichert (2025)). Intentionally overweighting such assets can result in an out-of-sample performance improvement (Hyung and de Vries (2005), Hwang et al. (2018), Yao et al. (2021), Kan and Lassance (2025)).

The remainder of the paper is organized as follows. Section 2 reviews existing portfolio construction methods and highlights the advantages of the dissimilarity heuristic. Section 3 evaluates out-of-sample performance, examines the statistical properties of the portfolios, and tests the robustness of the findings in a sector rotation setting. Section 4 demonstrates

that the dissimilarity heuristic maintains its desirable properties in basic asset allocation and domestic large-cap (individual stock return) settings. Section 5 concludes.

2 Portfolio Construction

2.1 Overview of the Existing Solutions

Existing long-only portfolio construction algorithms are designed to prioritize efficiency (the risk-return trade-off) over higher returns. As a result, investors have few viable options for increasing portfolio returns, and these often entail potentially unacceptable risks beyond mere increase in volatility. Altering the asset mix is perhaps the simplest approach. Choosing to invest exclusively in risky asset classes may raise the expected return of a portfolio, but not necessarily the compound return, while exposing the investor to the possibility of catastrophic loss due to overconcentration. Such behavior is associated with low levels of financial literacy and results in losses for those least capable of bearing them (Goetzmann and Kumar (2008), Anderson (2013), Gaudecker (2015)).

Mean-variance optimization suffers from both under-diversification and estimation error. The maximum return portfolio on an efficient frontier consists of a single asset. Selecting a portfolio along the efficient frontier between the maximum Sharpe ratio and the maximum return exchanges higher volatility for estimation error. Future returns are inherently difficult to forecast, and mean-variance optimization is highly sensitive to estimation errors in both expected returns and covariances. The resulting mean-variance optimal portfolios often exhibit extreme allocations to individual assets yet fail to deliver the expected high returns. Furthermore, such portfolios are sensitive to even small changes in expected returns, as the discrepancy between estimated and true optimal weights increases with the ratio of the largest to smallest eigenvalues of the covariance matrix (Best and Grauer (1991)), often leading to costly turnover during rebalancing. The extension of the mean-variance framework that allows investors to trade skewness for higher returns (de Athayde and Flores (2004)) remains largely theoretical and inherits all shortcomings of the original.

In practice, the 1/N rule is frequently used to address both underdiversification and parameter estimation error. DeMiguel et al. (2009) show that the 1/N rule performs better out-of-sample than naive mean-variance optimization and its extensions designed to minimize parameter estimation error. Jacobs et al. (2014) corroborate these conclusions, showing that "almost any form of naive fixed-weight allocation strategy realizes diversification potential." In fact, the 1/N rule approaches optimality under high model ambiguity (Pflug et al. (2012)) or when asset returns are largely driven by a single factor, with remaining risks being diversifiable (Yuan and Zhou (2024)).

Mixing strategies, which combine multiple portfolios to improve risk-adjusted returns, attempt to keep the desirable features of the 1/N rule while also incorporating additional information. Some of these strategies explicitly combine the 1/N rule with quantitative portfolio construction methods (Tu and Zhou (2011), Kircher and Rosch (2021), Lassance et al. (2024)). Other common mixing strategies incorporate adjustments for parameter uncertainty (Kan and Zhou (2007)), parameter uncertainty and fat tails (Kan and Lassance (2025)), or estimation risk (Golosnoy and Okhrin (2009), DeMiguel et al. (2013), Kan et al. (2022), Kan and Wang (2024)). These strategies usually do not seek to maximize returns; instead, they prioritize controlling various forms of risk, often reducing expected portfolio performance relative to the market. For example, Yuan and Zhou (2024) show that combining the 1/N rule with the global minimum variance portfolio maximizes the Sharpe ratio when the number of assets is small relative to the length of the historical return time series.

Other approaches, such as those based on cross-entropy, market timing, or incorporating non-asset return information, have been attempted, but either with limited success or targeting investment objectives different from the return maximization. Bera and Park (2008) demonstrate that minimizing cross-entropy addresses the estimation error but fail to offer statistically significant out-of-sample improvement compared to the 1/N rule. Farago and Hjalmarsson (2022) show that a frequently rebalanced portfolio with a relatively small number of assets can achieve superior long-run returns. When the number of assets exceeds the length of the time series, augmenting the 1/N rule with an "alpha portfolio" based on anomalies or

machine learning can lead to outperformance (Yuan and Zhou (2024)). However, Yuan and Zhou (2024) focus on maximizing the Sharpe ratio rather than targeting compound returns and their approach does not offer any "guarantee that the above combinations will perform as well in the future as they do now." Nevertheless, it remains unclear whether it is possible to systematically outperform the 1/N rule, preserve diversification, and include a large number of assets, all while relying exclusively on past returns.

2.2   Calculating Individual Asset Weights

I propose weighting assets in proportion to their dissimilarity from the rest of the portfolio, best illustrated with a simple, informal example. Suppose there are three assets, A, B, and C, with pairwise dissimilarities of 5 between A and B, 10 between A and C, and 15 between B and C. In this example, the weights of assets A, B, and C in the portfolio are:

$$w_A = 0.5 \cdot \frac{5 + 10}{5 + 10 + 15} = 0.250,$$
$$w_B = 0.5 \cdot \frac{5 + 15}{5 + 10 + 15} = 0.333,$$
$$w_C = 0.5 \cdot \frac{10 + 15}{5 + 10 + 15} = 0.417.$$

By design, asset C is the largest constituent of the portfolio because it differs most from assets A and B. The scaling factor of 0.5 accounts for the symmetry of asset pairs, since the dissimilarity between A and B is identical to that between B and A. Formally, for $N > 1$, where N is the number of assets, i and j index the assets, and $D(i, j)$ denotes the pairwise dissimilarity between assets i and j, the weight of asset i is given by:

$$w_i = 0.5 \cdot \frac{\sum_{j=1}^{N} D(i, j)}{\sum_{i=1}^{N} \sum_{j=1}^{N} D(i, j)}.$$

This formula has an intuitive interpretation. The numerator aggregates the pairwise dissimilarities between asset i and all other assets j, so assets that behave differently from the rest receive relatively higher weights. The denominator serves as a normalization term.

It represents the total dissimilarity across all asset pairs and ensures that the resulting portfolio weights sum to one. Consistent with Pflug et al. (2012), the weighting scheme also reduces to the 1/N rule when all pairwise dissimilarities are equal, reflecting high ambiguity or uncertainty. In essence, the method favors assets with distinct hedging behavior, while avoiding the explicit estimation of expected returns, covariance matrix, or its inverse.

2.3 Pairwise Dissimilarity Functions

The proposed portfolio construction method depends on the empirical measure of dissimilarity between assets. To this end, I rely on the two-sample Kolmogorov-Smirnov (KS) and Anderson-Darling (AD) test statistics, which measure the degree of difference between two empirical distributions. Both statistics have established theoretical properties, are computationally efficient, and have been used in a variety of financial applications.[1] Consider two samples of asset returns, $X_1, X_2, \ldots, X_m$ and $Y_1, Y_2, \ldots, Y_n$, with empirical cumulative distribution functions (ECDFs) $F_m(x)$ and $G_n(x)$, respectively. Then, the two-sample KS test statistic is defined as the maximum absolute difference between the two ECDFs:

$$KS_{m,n} = \sup_x |F_m(x) - G_n(x)|.$$

It measures the largest deviation between the distributions of the two samples, with larger values indicating greater dissimilarity. On the other hand, the two-sample Anderson-Darling (AD) test statistic is a weighted measure of the squared differences between the ECDFs. It is defined as:

$$AD_{m,n}^2 = \frac{mn}{m+n} \int_{-\infty}^{\infty} \frac{[F_m(x) - G_n(x)]^2}{H_{m+n}(x)\,[1 - H_{m+n}(x)]}\, dH_{m+n}(x),$$

where

$$H_{m+n}(x) = \frac{mF_m(x) + nG_n(x)}{m+n}$$

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[1]Proposed portfolio construction algorithm was implemented using two-sample KS and AD functions available in the R package "kSamples" (Scholz and Zhu (2023)).

is the pooled ECDF. Figure 1 provides a graphical illustration of these test statistics and highlights the differences between them.

Figure 1: **Measuring Dissimilarity: The Kolmogorov-Smirnov and Anderson-Darling Statistics**

This figure illustrates the difference between the Kolmogorov-Smirnov (KS) and Anderson-Darling (AD) test statistics. The KS statistic measures the maximum pointwise difference between two empirical cumulative distribution functions (ECDFs), while the two-sample AD statistic represents a weighted sum of the squared differences between the ECDFs computed at multiple points. By construction, the AD statistic places more weight on differences in the tails of the distributions, unlike the KS statistic, which is most sensitive near the center of the distribution.
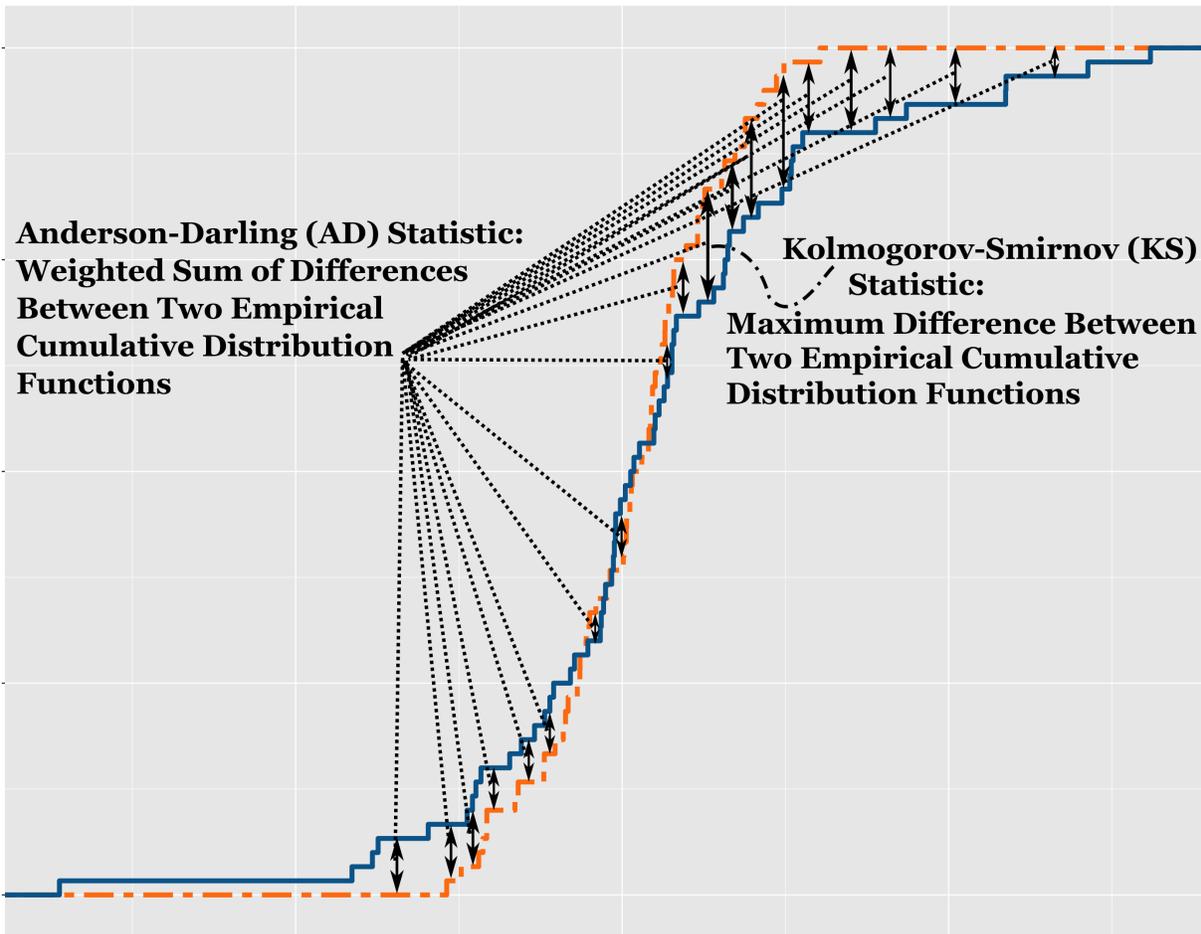
These nonparametric measures of empirical asset dissimilarity capture the entire return distribution rather than a finite number of moments, making them suitable for financial returns that are often skewed and exhibit heavy tails. Accumulating risks coming from the tails of the return distribution harnesses the compensation for the non-normality of returns and has the potential to substantially enhance out-of-sample performance (Hyung and de Vries (2005), Hwang et al. (2018), Yao et al. (2021), Kan and Lassance (2025)). Moreover, empirical evidence indicates that tail risks are priced, often at above-market rates and across various asset classes (Harvey and Siddique (2000), Kelly and Jiang (2014), Hua et al. (2021), Schreindorfer and Sichert (2025)).

The KS and AD measures also do not rely on any specific return-generating process or assumptions about potential underlying factors. When combined with the proposed weighting scheme, dissimilarity captures the portion of priced (reflected in observed returns) risks common to all assets, without requiring explicit assumptions about the quantity or required compensation for bearing those risks. Additionally, when an asset undergoes a rapid change in value, the method assigns progressively higher weights to it, thus directly capturing the momentum effect. Depending on the specific scenario and the portfolio's rebalancing frequency, this feature may contribute to both outperformance and underperformance.

This approach also has some limitations. When using KS or AD statistics, pairwise dissimilarity increases with differences in volatility, resulting in an asset exhibiting greater return fluctuations receiving a higher weight. From an implementation perspective, if the use of the KS or AD statistics is desired, it becomes the responsibility of the portfolio manager to curate the asset universe to ensure that only statistically suitable (mainly low volatility and low downside risk) assets are included. In this paper, the AD and KS statistics are selected primarily for their transparency, replicability, and computational tractability. However, a custom dissimilarity function could be defined to penalize low returns, high volatility, or incorporate subjective beliefs or investment ojectives. The development of such functions is left for future research.

3 Empirical Application: Sector Rotation Strategy

3.1 Portfolio Concentration and Asset Returns

To illustrate the practical viability of the proposed methodology, I implement a sector rotation strategy using 12 industry portfolios obtained from Ken French's data library. Individual industries have clearly defined risk premia and behave differently across market conditions. In such a setting, portfolio composition is expected to deviate substantially from the 1/N rule, leading to meaningful differences in investment performance. Additionally, sector rotation is a widely adopted investment strategy, with numerous providers offering separately managed accounts and exchange-traded products. For example, consider four popular actively managed fund-of-funds ETFs that implement sector or industry rotation: XLSR, THRO, SECT, and SSUS. Although these products are relatively new, they already accumulated over $11.2 billion in assets, with expense ratios ranging from 60 to 78 basis points (bps).[2] On the other hand, the underlying building blocks, individual sector and industry ETFs, offer substantially greater liquidity and significantly lower costs than the prepackaged funds-of-funds, with an average expense ratio of about 8 bps or lower.[3] As a result, from both institutional and retail perspectives, there are clear advantages to implementing a transparent, rules-based weighting scheme rather than paying for active management that often fails to generate excess returns.

Table 1 displays summary statistics for individual industry returns over 2015-2024 (120 monthly value-weighted observations). This time frame is particularly notable because it encompasses a range of distinct market environments: moderate expansion, rapid growth, and the COVID-19 recession, which resulted in a sharp market decline and temporarily reshaped the relative economic importance of various industries. It also features periods of rising, falling, and near-zero interest rates, capturing substantial variation in monetary policy. Across industries, arithmetic mean returns range from 5.99% to 23.14%, while volatility spans 13.37% to 38.30%. Cumulative annualized returns also exhibit substantial heterogeneity, varying from 4.59% to 20.24%. This heterogeneity in industry returns is advantageous;

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[2]As of 12/31/2025.

[3]Such as SPDR, iShares, Vanguard or Fidelity individual Sector and Industry ETFs.

when used as a training sample, the variation can expose flaws in the portfolio construction methodology and indicate its shortcomings.

Table 1: **Summary Statistics: Industry Returns**

This table reports summary statistics for individual industry returns over the 2015-2024 period (10 years), based on 120 monthly value-weighted observations. The mean, standard deviation, and geometric mean (GeoM) are annualized. Min, P25, P50, P75, and Max represent the minimum, 25th, 50th, and 75th percentiles, and the maximum of monthly returns, respectively. The simple Sharpe ratio (SR) is calculated as the mean return divided by the standard deviation. All reported values (except for the Sharpe ratio) are expressed in percent.

	Mean	SD	GeoM	SR	Min	P25	P50	P75	Max
Nondurables	7.04	13.37	6.32	0.53	−11.38	−1.56	0.49	3.24	10.02
Durables	23.14	38.30	17.53	0.60	−28.07	−4.40	0.43	6.27	40.99
Manufacturing	12.04	19.82	10.56	0.61	−20.30	−2.20	0.78	4.24	17.06
Energy	9.55	30.33	5.10	0.31	−34.47	−3.43	0.51	4.70	32.36
Chemicals	7.33	15.46	6.32	0.47	−10.75	−2.20	0.83	3.10	12.12
Bus. Eqpt.	20.34	18.90	20.24	1.08	−12.60	−1.68	1.86	5.46	14.98
Communication	5.99	17.35	4.59	0.35	−13.83	−2.48	0.46	3.64	14.48
Utilities	8.73	14.79	7.91	0.59	−12.94	−1.95	1.10	3.57	10.37
Shops	14.73	16.88	14.19	0.87	−9.78	−1.56	1.08	3.78	18.17
Healthcare	9.32	15.20	8.48	0.61	−10.13	−1.93	0.80	4.13	13.48
Finance	13.56	19.61	12.27	0.69	−20.18	−2.36	1.89	4.80	16.32
Other	11.81	18.55	10.57	0.64	−17.71	−2.04	0.90	4.03	15.00

Table 2 shows allocations produced by different portfolio construction methods. Regardless of the dissimilarity measure, assets with higher standard deviations receive larger weights. The highest weights under the KS and AD statistics, 11.16% and 18.00% respectively, correspond to Durables. This industry also has the highest standard deviation among all industries at 38.30% and the second highest geometric mean at 17.53%. It is worth noting that the high weight on Durables is not a result of Sharpe ratio maximization. The Sharpe ratio of Durables, 0.60, ranks only 7th out of 12 industries. The pairwise dissimilarity heuristic does not explicitly target maximizing the geometric mean return either; high-risk yet low-return industries also receive above-average portfolio weights. For example, Energy has the second-lowest geometric mean return in this sample, at 5.10%, and the lowest Sharpe ratio of 0.31 and yet it constitutes 9.05% and 11.29% of the KS and AD portfolios respectively.

Constructing a portfolio based on pairwise dissimilarity does not lead to overly concentrated positions in some assets or zero allocations to others. With the KS function, asset weights range from 6.84% to 11.16%, and with the AD function, they range from 5.05% to 18.00%. By construction, no weight is zero, and no single asset dominates the portfolio. On the other hand, the long-only minimum-variance portfolio constructed using the sample covariance matrix holds only three of the twelve available assets: Nondurables (37.26%), Utilities (32.10%), and Healthcare (30.64%). The 1/N allocation is 8.33%, but it is noninformative and ignores all information contained in the returns. Nevertheless, the 1/N rule remains a suitable benchmark since it is robust to the estimation error and is often used as either the shrinkage target or initial portfolio when constructing a risk-managed allocation.

Table 2: **Portfolio Construction Methodology and Asset Weights**

This table reports asset weights for portfolios constructed using individual industry returns over the 2015-2024 period (10 years, 120 monthly value-weighted observations). 1/N denotes the equally weighted portfolio, and MVP stands for the long-only minimum-variance portfolio constructed using the sample covariance matrix. ΣD is the sum of pairwise differences between a given asset and all other assets, computed using the Kolmogorov-Smirnov (KS) or Anderson-Darling (AD) statistics, and w is the corresponding weight. All reported values (except for ΣD) are in percent.

	KS		AD		1/N	MVP
	ΣD	w	ΣD	w	w	w
Nondurables	1.56	9.62	27.43	11.22	8.33	37.26
Durables	1.81	11.16	44.00	18.00	8.33	0
Manufacturing	1.17	7.20	12.34	5.05	8.33	0
Energy	1.47	9.05	27.59	11.29	8.33	0
Chemicals	1.34	8.28	17.22	7.05	8.33	0
Bus. Eqpt.	1.57	9.67	25.73	10.53	8.33	0
Communication	1.22	7.51	14.11	5.77	8.33	0
Utilities	1.23	7.61	17.82	7.29	8.33	32.10
Shops	1.28	7.87	15.30	6.26	8.33	0
Healthcare	1.25	7.72	16.18	6.62	8.33	30.64
Finance	1.21	7.46	14.84	6.07	8.33	0
Other	1.11	6.84	11.88	4.86	8.33	0

By design, pairwise dissimilarity allocations depend on the underlying comovement among assets. The numerator (ΣD) of the weighting scheme is the sum of pairwise differences between

an asset and the remaining portfolio constituents, so assets that are more dissimilar from the rest receive relatively higher weights. For example, consider the Durables-Nondurables pair. Individually, ΣD_{KS} for Durables is 1.81 and ΣD_{AD} is 44.00, the highest among all assets. For Nondurables, the corresponding ΣD_{KS} and ΣD_{AD} values are 1.56 and 27.43, respectively – the third-highest among all ΣD for both KS and AD statistics. This relationship is likely due to the embedded macroeconomic patterns, captured by the dissimilarity heuristic. Consumers purchase durable and nondurable goods at the opposite stages of the business cycle, so holding both in a portfolio provides a source of return across a wide range of economic conditions. As expected, the opposite relationship holds for low ΣD values. Among the industries, Finance has the third-highest Sharpe ratio of 0.69, but a below-average ΣD_{KS} of 1.21 and ΣD_{AD} of 14.84. Its portfolio weight is 7.46% under the KS statistic and 6.07% under the AD statistic. The corresponding 1/N allocation is 8.33%, and Finance is not in the minimum-variance portfolio. However, Finance provides services such as borrowing, lending, and transaction processing that are essential to all industries, which helps explain its limited appeal to the dissimilarity heuristic. Despite this, its weight is only slightly lower than that of other assets and is not reduced to zero, so the dissimilarity portfolios retain the diversification characteristics of the 1/N rule.

3.2   Out-of-Sample Portfolio Performance

The main objective for the proposed dissimilarity heuristic is to achieve higher risk and higher returns while maintaining diversification, volatility, tail risk, and turnover at levels comparable to the baseline 1/N rule. To establish an initial comparison with the 1/N benchmark, I again consider the 2015-2024 period but this time as a testing sample. When used as a testing sample, the variation in individual asset returns causes portfolio performance to depend heavily on the construction method, resulting in meaningful differences in investment outcomes.

Table 3: **Out-of-Sample Portfolio Performance**

This table shows out-of-sample portfolio performance over 2015-2024. Panel A presents performance statistics for portfolio weights calculated using monthly data and rebalanced each month. Panel B presents performance statistics for portfolio weights calculated using monthly data and rebalanced yearly. Panel C presents performance statistics for portfolio weights calculated using daily data and rebalanced monthly. Panel D presents performance statistics for portfolio weights calculated using daily data and rebalanced yearly. The portfolios are constructed using 12 value-weighted industry indices. The estimation window is set to a fixed period of 10 calendar years (120 monthly returns or 10*12*21≈2,520 daily returns, depending on the actual year). The mean, standard deviation, geometric mean (GeoM), and Sharpe ratio (SR) are annualized. Mean, standard deviation, geometric mean, maximum drawdown (Max Drawd.), Value-at-Risk, and Conditional Value-at-Risk are expressed as percentages (i.e., 11.96 stands for 11.96%). Upside (Up. Capt.) and downside (Down. Capt.) capture ratios are calculated relative to the corresponding 1/N portfolio.

	Mean	SD	GeoM	SR	Max Drawd.	Skew.	Kurt.	Up. Capt.	Down. Capt.	VaR (95%)	CVaR (95%)
Panel A: Monthly Data, Monthly Rebalancing											
KS	12.27	16.32	11.51	0.65	24.33	−0.17	1.65	1.01	1.00	−6.76	−10.28
AD	12.93	16.96	12.14	0.66	25.36	−0.09	2.03	1.03	1.00	−6.86	−10.52
1/N	11.96	16.01	11.23	0.64	23.21	−0.20	1.32	1	1	−6.71	−9.99
Panel B: Monthly Data, Yearly Rebalancing											
KS	12.85	16.40	12.15	0.68	23.90	−0.15	1.44	1.01	1.00	−6.74	−10.08
AD	13.74	17.05	13.02	0.71	24.69	−0.05	1.70	1.05	1.00	−6.82	−10.20
1/N	12.50	16.09	11.81	0.67	22.85	−0.16	1.21	1	1	−6.66	−9.79
Panel C: Daily Data, Monthly Rebalancing											
KS	12.44	16.82	11.62	0.64	24.82	−0.11	1.75	1.03	1.02	−6.90	−10.43
AD	12.89	17.74	11.94	0.63	26.50	−0.02	2.15	1.05	1.03	−7.12	−10.78
1/N	11.96	16.01	11.23	0.64	23.21	−0.20	1.32	1	1	−6.71	−9.99
Panel D: Daily Data, Yearly Rebalancing											
KS	13.25	16.98	12.49	0.68	24.32	−0.05	1.57	1.04	1.02	−6.84	−10.10
AD	13.93	17.93	13.07	0.68	25.84	0.05	1.91	1.07	1.04	−7.04	−10.35
1/N	12.50	16.09	11.81	0.67	22.85	−0.16	1.21	1	1	−6.66	−9.79

Table 3 shows out-of-sample portfolio performance over 2015-2024, with the estimation window set to a fixed period of 10 calendar years (120 monthly returns or $10*12*21 \approx 2,520$ daily returns, depending on the actual year). The table presents results grouped by estimation and rebalancing frequencies. Panel A reports performance statistics for monthly rebalanced portfolios with weights estimated from monthly data. Both the arithmetic (labeled Mean) and geometric (GeoM) average returns are higher for the dissimilarity strategy than for the 1/N rule. However, in this setting, the geometric mean return is a more appropriate statistic

for comparing portfolio performance because it accounts for return volatility and reflects actual annualized investment growth. Using the KS statistic, the difference in geometric mean returns is $11.51 - 11.23 = 28$ bps. Using the AD statistic, the outperformance is $12.14 - 11.23 = 91$ bps. The outperformance of 28 to 91 basis points is economically significant. For example, consider a worker who contributes \$10,000 per year to a defined contribution plan and works for 30 years. If his investments earn a conservative average annual return of 6%, then at retirement he would have approximately:

$$FV \approx 10{,}000 \cdot \frac{(1.06)^{30} - 1}{0.06} \approx 10{,}000 \cdot 79.06 = 790{,}600.$$

If instead he adopts a more aggressive allocation that delivers an average annual return of 6.6 percent, corresponding to the midpoint of the 28 to 91 basis point range, then the final account balance would become 879,300. A difference of \$88,700, representing an 11.22% increase in lifetime savings, is economically meaningful.

The additional return associated with the pairwise dissimilarity approach is achieved without materially increasing risk. The standard deviations of the KS, AD, and 1/N strategies are 16.32, 16.96, and 16.01 percent respectively, resulting in statistically indistinguishable Sharpe ratios of 0.65 (KS), 0.66 (AD), and 0.64 (1/N). These Sharpe ratios are remarkable for a return-seeking portfolio. The CAPM penalizes risk-taking, making it difficult to achieve higher returns without compromising the risk-return tradeoff (Black (1972), Fama and French (2004)). Risk is not shifted to the tail either: maximum drawdown, skewness, kurtosis, VaR, and CVaR are all comparable to the 1/N baseline. This result demonstrates that pairwise dissimilarity has at least some risk mitigation properties; in a similar setting, Candes et al. (2025) found that "thematic baskets with statistically significant average pairwise correlation will have risk levels above predictions." Additionally, upside and downside capture ratios provide intuition for the timing of additional return: because of its more aggressive allocation, the dissimilarity-based portfolio tends to earn higher returns in up markets without incurring proportional losses in down markets.

Table 3, Panel B examines how the performance of the dissimilarity-based heuristic depends

on rebalancing frequency. Instead of rebalancing every month (Panel A), the portfolio is rebalanced annually (Panel B). Under annual rebalancing, the pairwise dissimilarity approach outperforms the 1/N rule even more. The differences in geometric mean returns are 12.15 – 11.81 = 34 bps and 13.02 – 11.81 = 121 bps for the KS and AD statistics, respectively. The Sharpe ratio of the dissimilarity portfolio constructed using the AD statistic (0.71) is noticeably higher than that of the 1/N portfolio (0.67), while the KS (0.68) is in line. The outperformance comes from capturing the momentum effect. When an asset experiences a rapid change in value, the dissimilarity heuristic assigns progressively higher weights to it and adjusts them less frequently. This finding is consistent with Candes et al. (2025), who find that "investment strategies based on trends identified before release might persist after release, creating an alpha opportunity." Moreover, the primary applications of this methodology are long-term savings, strategic asset allocation, and indexing. These are all settings in which portfolios are generally rebalanced less frequently and stand to benefit from capturing positive trending.

Table 3, Panels C and D explore the potential dependence on data frequency. Daily returns are more volatile than monthly returns and typically exhibit heavier tails, which may affect portfolio weights and investment performance. Consistent with daily returns being more heavy-tailed and the AD statistic's sensitivity to the tails of the distribution, a comparison of geometric means in Panels C and D with those in Panels A and B shows that KS-based dissimilarity portfolios benefit from using daily data 11.62 > 11.51, 12.49 > 12.15), whereas the effect on AD-based portfolios is mixed (11.94 < 12.14, 13.07 > 13.02). However, holding rebalancing and estimation frequencies fixed, both the KS and AD dissimilarity portfolios continue to outperform the 1/N rule (11.62, 11.94 > 11.23; 12.49, 13.07 > 11.81).

Table 4 examines whether the additional performance of the dissimilarity heuristic is achieved at the cost of increased trading. It reports one-way turnover measured at the portfolio level and at the individual position level, with the key evaluation metrics being average portfolio turnover and maximum individual position turnover. To illustrate these metrics, consider rebalancing a portfolio from a 70-10-20 allocation to a 60-40-0. It results in

one-way turnover of $(10 + 30 + 20)/2 = 30\%$. For the same 70-10-20 to 60-40-0 trade, the maximum turnover across individual positions is $max(10, 30, 20) = 30\%$. Average portfolio turnover largely determines total trading costs, while maximum turnover across individual positions reflects potential market impact. All turnover statistics are computed both with and without accounting for portfolio drift.

Table 4: **Portfolio Turnover**

This table shows turnover statistics over 2015-2024. Panel A demonstrates the difference between new and old allocations constructed using monthly data; reported statistics exclude the effects of portfolio drift due to market movements. Panel B is identical to the Panel A, but includes the effects of portfolio drift. Panels C and D correspond to the Panels A and B respectively, but the allocations are constructed using daily data. Panels that include drift (B and D) reflect the actual trading required to implement a new allocation. The estimation window is set to a fixed period of 10 calendar years (120 monthly returns or $10 * 12 * 21 \approx 2,520$ daily returns, depending on the actual year). The portfolios are rebalanced monthly. All statistics are expressed as percentages (i.e., 0.22 stands for 0.22%) and are reported on a per-reallocation basis (not annualized). Values less than 0.0001 are reported as 0.

	Portfolio				Individual Positions			
	Min	**Mean**	**Max**	**SD**	**Min**	**Mean**	**Max**	**SD**
Panel A: Monthly Data, Without Drift								
KS	0.22	0.66	1.32	0.18	0	0.11	0.68	0.10
AD	0.41	1.04	2.27	0.34	0.0001	0.17	1.38	0.18
1/N	0	0	0	0	0	0	0	0
Panel B: Monthly Data, With Drift								
KS	0.49	1.56	4.20	0.68	0.0004	0.26	2.78	0.31
AD	0.64	1.86	4.60	0.88	0	0.31	3.79	0.44
1/N	0.41	1.30	3.75	0.62	0.0001	0.22	2.55	0.24
Panel C: Daily Data, Without Drift								
KS	0.08	0.30	0.49	0.08	0	0.05	0.28	0.05
AD	0.13	0.36	0.74	0.13	0	0.06	0.63	0.07
1/N	0	0	0	0	0	0	0	0
Panel D: Daily Data, With Drift								
KS	0.44	1.48	4.06	0.77	0	0.25	3.53	0.34
AD	0.40	1.64	4.66	0.96	0	0.27	4.62	0.45
1/N	0.41	1.30	3.75	0.62	0.0001	0.22	2.55	0.24

Table 4, Panel A shows that shifting the estimation window by one month results in an average allocation change of 0.66% for the KS portfolio and 1.04% for the AD portfolio. This difference arises from higher maximum trading at the individual position level (0.68% vs.

1.38%), but remains low and is unlikely to pose problems even for positions with limited liquidity. Panel B accounts for the effects of portfolio drift. Average portfolio turnover is nearly identical across the dissimilarity portfolios and the 1/N benchmark (1.56%, 1.86%, and 1.30%). Compared with the 1/N benchmark, maximum individual position turnover is similar for the KS portfolio (2.78% vs. 2.55%) and only slightly higher for the AD allocation (3.79% vs. 2.55%). Panels C and D use daily data to address concerns that the relatively low turnover in Panels A and B may be an artifact of using monthly returns. With drift, after switching from monthly to daily returns for estimation, maximum one-position turnover increases by about 1%, rising from 2.78% to 3.53% for KS portfolios and from 3.79% to 4.62% for AD portfolios. The average trading remains largely unaffected. Overall, the dissimilarity heuristic exhibits slightly higher turnover than the 1/N rule, but the difference is too small to offset its performance advantage.

3.3 Robustness of Dissimilarity Heuristic

This section demonstrates that the dissimilarity heuristic is robust to practical implementation concerns, including sensitivity to estimation parameters, dependence on market timing, and estimation error. The strategy's outperformance relative to the 1/N rule persists and does not appear to be an artifact of parameter choices or sampling variation.

Table 5 shows the relationship between out-of-sample portfolio performance, turnover, and the length of the estimation window. The evaluation period remains the same, spanning 2015–2024, while the estimation window varies from 12 to 180 months. When the KS statistic is used for dissimilarity-based portfolio construction, geometric mean returns range from 11.11% to 11.74%, closely matching or outperforming the 1/N baseline of 11.23%. Using the AD statistic, geometric mean returns range from 11.40% to 12.60%, outperforming the 1/N portfolio across all estimation windows. This outperformance also does not come at the expense of the risk-return trade-off or turnover. Sharpe ratios for both the KS (0.63–0.66) and AD (0.62–0.68) specifications are comparable to, and often exceed, that of the 1/N portfolio (0.64). Turnover remains broadly in line with the 1/N strategy: for estimation windows of

48 months and longer, turnover is only slightly higher than that of the 1/N portfolio and declines as the estimation window length increases.

Table 5: **Sensitivity of Portfolio Performance to Estimation Window Length**

This table shows the relationship between out-of-sample portfolio performance, turnover, and estimation window length. The estimation window length (Window), indicates the number of monthly returns used to estimate the asset weights. Portfolios are constructed using 12 monthly value-weighted industry indices, with the evaluation period spanning 2015-2024. The portfolios are rebalanced monthly. The mean, standard deviation, geometric mean (GeoM), and Sharpe ratio (SR) are annualized. Mean portfolio turnover (Turn.) is reported on a per-reallocation basis, accounts for the market drift, and reflects the actual trading required to implement a new set of weights. All statistics except for the Sharpe ratio are expressed as percentages (i.e., 11.96 stands for 11.96%).

Window	KS					AD				
	Mean	SD	GeoM	SR	Turn.	Mean	SD	GeoM	SR	Turn.
12	12.50	16.51	11.74	0.66	3.41	13.37	17.09	12.60	0.68	5.00
24	12.28	16.47	11.50	0.64	2.43	12.82	17.21	11.97	0.65	3.57
36	12.18	16.34	11.42	0.64	2.10	12.79	17.09	11.96	0.65	3.03
48	11.91	16.34	11.11	0.63	1.82	12.41	17.06	11.54	0.63	2.59
60	11.97	16.44	11.16	0.63	1.77	12.36	17.22	11.45	0.62	2.40
72	12.00	16.47	11.19	0.63	1.72	12.32	17.24	11.40	0.62	2.28
84	12.21	16.39	11.44	0.64	1.67	12.73	17.10	11.89	0.65	2.15
96	12.24	16.40	11.47	0.64	1.62	12.71	17.10	11.86	0.64	2.02
108	12.26	16.38	11.49	0.65	1.60	12.89	17.04	12.07	0.66	1.95
120	12.27	16.32	11.51	0.65	1.56	12.93	16.96	12.14	0.66	1.86
132	12.26	16.32	11.50	0.65	1.53	12.93	16.94	12.14	0.66	1.82
144	12.25	16.30	11.49	0.65	1.51	12.84	16.89	12.05	0.66	1.78
156	12.19	16.27	11.43	0.65	1.49	12.64	16.92	11.83	0.65	1.74
168	12.10	16.29	11.33	0.64	1.47	12.48	16.94	11.64	0.64	1.72
180	12.16	16.28	11.40	0.64	1.48	12.61	16.94	11.78	0.64	1.71
1/N	11.96	16.01	11.23	0.64	1.30	11.96	16.01	11.23	0.64	1.30

Table 6 investigates the dependence of portfolio performance on market timing. Portfolios are formed at the beginning of each month from 1999 to 2023 and evaluated out-of-sample over the subsequent 12 months, resulting in $12 * 25 = 300$ distinct starting points. Sector rotation represents a particularly conservative setting, as short-term industry returns are largely zero-sum and the benefits typically accrue only over longer horizons. Additionally, market-wide gains or losses common to all industries are also captured by the benchmark 1/N portfolio. Nevertheless, over a one-month evaluation horizon, the dissimilarity strategies based on both the KS and AD statistics slightly outperform the 1/N rule (0.776% and 0.778%

vs. 0.774%), indicating that the methodology does not rely on favorable short-term timing and is unlikely to underperform systematically. When portfolios are held for one year, the cumulative outperformance of the KS strategy is 6 bps (10.21% vs. 10.15%), while the AD strategy outperforms by 18 bps (10.33% vs. 10.15%), with Sharpe ratios remaining nearly identical to those of the 1/N benchmark. Annual turnover for the KS and AD strategies is 5.68% and 7.07%, respectively, only modestly higher than that of the 1/N rule (4.62%) and unlikely to negate the observed performance advantages. Finally, Figure 2 plots dissimilarity strategy outperformance relative to the 1/N rule (y-axis) against turnover (x-axis). There is no clear observable relationship, indicating that higher (lower) turnover does not correspond to higher (lower) performance. This observation further suggests that trading activity and illiquidity are unlikely to be the primary drivers of the dissimilarity heuristic's outperformance.

Table 6: **Market Timing and Short-Term Portfolio Performance**

This table shows short-term portfolio performance and its dependence on the market timing over 1999-2024. Portfolios are constructed using 12 monthly value-weighted industry indices. The estimation window is set to a fixed period of 10 calendar years (120 monthly returns). Portfolios are constructed at the beginning of each month from 1999 to 2023 and evaluated out-of-sample for the next 12 months, resulting in $12 * 25 = 300$ distinct starting points. 1M is the arithmetic average of 300 out-of-sample monthly returns immediately following each starting point. Yearly statistics (12M) are averages of 300 observations, with each observation computed using 12 out-of-sample monthly returns. Yearly statistics include annualized mean (Mean), annualized standard deviation (SD), annualized Sharpe ratio (SR), cumulative return (Cumulative), and portfolio turnover (Turn.). Cumulative return is a total percentage change in the value of a portfolio obtained by chain-linking 12 individual monthly out-of-sample returns. Portfolio turnover (Turn.) is calculated with respect to the similarly constructed portfolio at the end of the out-of-sample evaluation, accounts for the market drift, and reflects the actual trading required to implement a new set of weights. All statistics except for the Sharpe ratio are expressed as percentages (i.e., 10.15 stands for 10.15%).

| | | 12M | | | | |
	1M	Mean	SD	Cumulative	SR	Turn.
KS	0.776	9.73	14.12	10.21	0.836	5.68
AD	0.778	9.84	14.59	10.33	0.817	7.07
1/N	0.774	9.66	13.93	10.15	0.839	4.62

Lastly, I examine the robustness of the proposed dissimilarity heuristic to estimation error. From an empirical perspective, estimation error represents a major challenge in portfolio construction, frequently offsetting the gains from sophisticated algorithms and limiting their practical applicability. I sample normally distributed noise from $\mathcal{N}(0, k*SD_i)$ distribution

and then add it to the estimation data. SD_i is the standard deviation of the ith asset, and k is a scalar that controls the level of simulated noise. For each combination of starting date and noise level, the noise is drawn 100 times, resulting in $25 * 10 * 100 = 25,000$ distinct simulations. I assess the impact on the portfolio in two ways: first, by measuring the turnover required to adjust noisy asset weights to those estimated without added noise; and second, by examining whether the added noise materially affects subsequent performance.

Figure 2: **Excess Turnover and Strategy Outperformance**

This figure displays 300 pairwise combinations of excess turnover and outperformance over 1999-2024. Excess portfolio turnover (x-axis) is defined as the difference between the strategy turnover (KS or AD) and the corresponding 1/N portfolio turnover. Outperformance (y-axis) is defined as the difference between the 12-month cumulative return (KS or AD dissimilarity strategy) and the corresponding return on a 1/N portfolio. Dashed line is the OLS estimate of the relationship between excess turnover and outperformance. Portfolios are constructed at the beginning of each month from 1999 to 2023 using 12 monthly value-weighted industry indices and evaluated out-of-sample for the next 12 months, resulting in 12*25=300 distinct observations. The estimation window is set to a fixed period of 10 calendar years (120 monthly returns).

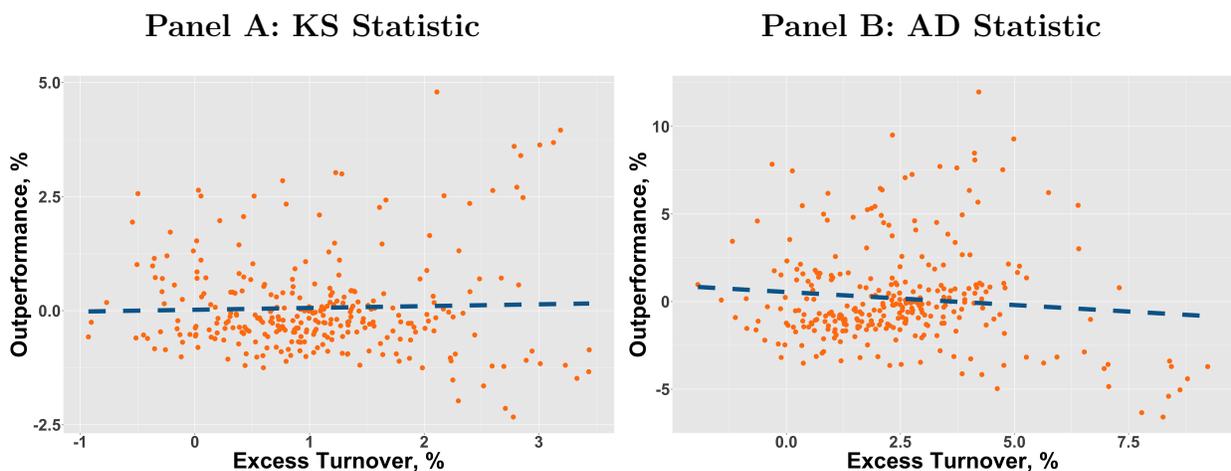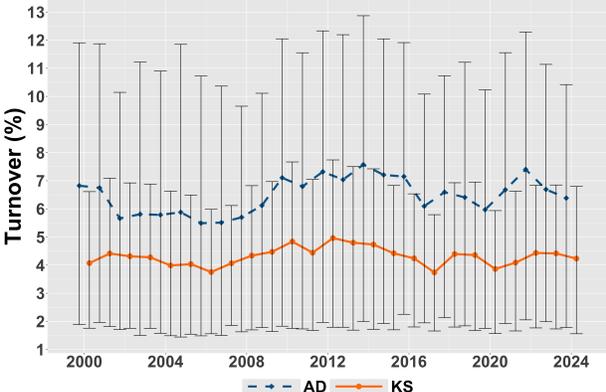Panel A: KS Statistic   Panel B: AD Statistic

Table 7 shows the results. Under a low estimation error scenario (k=0.1), the KS and AD portfolios require average turnover of 1.77% and 1.87%, respectively, to adjust noisy weights to their noise-free counterparts. These turnover levels are comparable to those induced by incorporating one additional month of data. Even at the upper tail, the 95th percentile turnover is limited to 2.51% for the KS portfolio and 2.78% for the AD portfolio, indicating that both statistics exhibit substantial robustness to estimation error. However, as the level of estimation error increases, AD portfolios become relatively more sensitive

to noise, with average turnover eventually rising to 9.10% compared to 5.59% for the KS portfolios. Moreover, unlike the KS portfolios, turnover for the AD portfolios does not taper off around $k=0.6$ and continues to increase. Interestingly, even with added noise, both the KS and AD portfolios on average slightly outperform their noise-free counterparts, with mean outperformance reaching at most 15 bps suggesting that estimation error has limited impact on portfolio performance. Furthermore, the 95th percentile outcomes slightly exceed the 5th percentile, suggesting that the heuristic tends to gain rather than lose from extreme events, without materially mitigating their impact. This interpretation is supported by Figure 3, which plots the impact of estimation error over time. Average and extreme turnover show little dependence on time (Panel A), whereas the magnitude of performance impact is largest around major market events, including the dot-com bubble, the 2008 global financial crisis, and the COVID-19 pandemic (Panel B).

Figure 3: **Impact of Estimation Error by Year**

This figure displays the results of using Monte-Carlo simulations (N=25,000) to generate estimation error and assess its impact on individual asset weights (Panel A) and portfolio performance (Panel B) by year (x-axis). The impact on portfolio weights is measured by the turnover required to adjust noisy asset weights to those estimated without the simulated noise. Portfolio outperformance is defined as the difference between the noisy portfolio's 12-month cumulative return and the corresponding return of a portfolio estimated without the simulated noise. Positive (negative) values indicate better (worse) cumulative returns for portfolios estimated with noise. Lines and markers display average values of turnover (Panel A) and outperformance (Panel B). KS strategy is drawn using a solid line with a round marker; AD strategy uses dashed line with a diamond marker. Whiskers reflect 5th and 95th percentiles. Horizontal jitter is added for clarity.

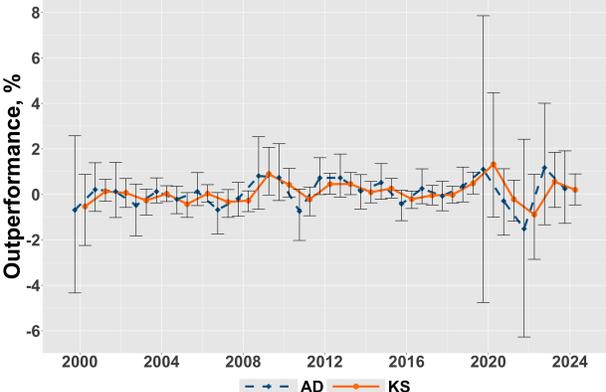**Panel A: KS Statistic**　　　　　　**Panel B: AD Statistic**

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Table 7: **Impact of Estimation Error**

This table shows the results of using Monte Carlo simulations to generate estimation error and assess its impact on individual asset weights and portfolio performance. Portfolios are formed at the start of each year from 1999 to 2023 using monthly value-weighted industry indices and are evaluated out-of-sample over the subsequent 12 months. The estimation window is 10 years (120 monthly returns), with normally distributed noise, $\mathcal{N}(0, k*SD_i)$, added to the estimation data. SD_i denotes the standard deviation of the ith asset, and k is a scalar controlling the level of simulated noise. For each date-level combination, noise is drawn 100 times, yielding $25 * 10 * 100 = 25,000$ distinct simulations. The impact on portfolio weights is measured by the turnover (labeled Allocations Turnover) required to adjust noisy asset weights to those estimated without any noise added. Portfolio performance (labeled Portfolio Performance) is defined as the difference between the noisy portfolio's 12-month cumulative return and the corresponding return of a portfolio estimated without any noise added. Positive (negative) values indicate better (worse) cumulative returns for portfolios estimated with noise. 5% and 95% refer to the respective percentiles; Avg. stands for the average. All statistics are expressed as percentages (e.g., 0.15 denotes 0.15%).

| | Allocations Turnover | | | | | | Portfolio Performance (12M) | | | | | |
| | KS | | | AD | | | KS | | | AD | | |
k	5%	Avg.	95%	5%	Avg.	95%	5%	Avg.	95%	5%	Avg.	95%
0.1	1.11	1.77	2.51	1.10	1.87	2.78	−0.33	0.03	0.40	−0.49	0.00	0.48
0.2	1.77	2.80	3.91	2.03	3.48	5.27	−0.54	0.04	0.61	−0.89	0.02	0.93
0.3	2.19	3.50	4.89	2.77	4.88	7.35	−0.73	0.07	0.87	−1.24	0.03	1.48
0.4	2.57	4.10	5.74	3.50	6.00	8.95	−0.83	0.09	1.12	−1.51	0.04	1.67
0.5	2.90	4.55	6.35	3.88	6.78	9.95	−0.91	0.07	1.14	−1.68	0.10	1.97
0.6	3.08	4.88	6.80	4.37	7.47	10.99	−1.05	0.08	1.24	−1.95	0.10	2.05
0.7	3.27	5.09	7.10	4.79	8.01	11.68	−1.06	0.08	1.25	−1.88	0.15	2.23
0.8	3.44	5.30	7.26	4.81	8.39	12.49	−1.08	0.10	1.37	−2.01	0.11	2.10
0.9	3.48	5.50	7.57	5.20	8.79	12.77	−1.14	0.10	1.34	−2.22	0.11	2.33
1.0	3.60	5.59	7.85	5.42	9.10	13.20	−1.15	0.11	1.47	−2.36	0.15	2.48

4 Validation

4.1 Basic Asset Allocation

Asset allocation is another application where portfolio construction choice leads to meaningfully different investment outcomes. Additionally, asset allocation presents a distinct set of challenges, largely driven by often extreme heterogeneity in returns across asset classes, causing even simple weighting schemes to produce overly concentrated portfolios. For example, Chow et al. (2014) introduce two heuristic weighting schemes, inverse volatility and inverse beta. These schemes use the same weighting method as in this paper, but rely on asset volatil-

ity and CAPM beta instead of dissimilarity. Now consider the most basic two-asset portfolio, consisting of short-term Treasuries (proxied by SHV, StDev $\approx 0.2\%$ as of 12/31/2025) and a fund tracking the S&P 500 (proxied by IVV, StDev $\approx 12\%$ as of 12/31/2025). Under inverse volatility weighting, the resulting portfolio weights are $(1/0.002)/(1/0.002 + 1/0.12) = 98.36\%$ and 1.64% respectively. Moreover, replacing the standard deviation with variance would result in a different set of portfolio weights, despite both measures capturing the same underlying volatility. A similar issue arises for inverse beta weighting. Selecting an asset with a negative beta or near-zero beta may lead to extreme and practically implausible allocations. The dissimilarity heuristic does not suffer from these issues and therefore represents a useful and complementary addition to the portfolio construction toolkit.

Table 8: **Portfolio Performance and Turnover: Asset Allocation Setting**

This table shows the out-of-sample portfolio performance and turnover statistics for the maximum pairwise dissimilarity portfolios in an asset allocation setting. Portfolios are constructed using 5 highly liquid ETFs each corresponding to a distinct asset class. They are SPY (U.S. Equities), VNQ (U.S. REITs), EFA (Developed Market Equities), TLT (Long Treasuries), GLD (Gold, Liquid Alternatives). The portfolios are rebalanced either monthly (Panel A) or yearly (Panel B) and the evaluation period spans 2015-2024. The estimation period is set to 10 years (120 monthly returns). The mean, standard deviation, geometric mean (GeoM), and Sharpe ratio (SR) are annualized. Minimum, mean, and maximum portfolio turnover statistics are reported on a per-reallocation basis, account for the market drift, and reflect the actual trading required to implement a new set of weights. VaR and CVaR stand for 95% Value-at-Risk, and Conditional Value-at-Risk respectively. All statistics except for the Sharpe ratio are expressed as percentages (i.e., 6.75 stands for 6.75%).

	Mean	SD	GeoM	SR	VaR (95%)	CVaR (95%)	Turnover Min	Turnover Mean	Turnover Max
				Panel A: Monthly Rebalancing					
KS	7.26	11.03	6.86	0.51	−4.73	−6.23	0.17	1.39	4.99
AD	7.16	10.99	6.76	0.50	−4.70	−6.13	0.29	1.73	5.10
1/N	6.94	11.17	6.51	0.47	−4.83	−6.41	0.29	1.18	4.79
				Panel B: Yearly Rebalancing					
KS	7.03	10.82	6.65	0.50	−4.63	−6.01	0.56	4.14	8.30
AD	6.95	10.72	6.57	0.49	−4.57	−5.88	1.64	5.35	11.01
1/N	6.75	10.99	6.33	0.46	−4.75	−6.22	1.78	3.55	7.07

Table 8 displays the out-of-sample portfolio performance and turnover statistics for the maximum pairwise dissimilarity portfolios in an asset allocation setting. Portfolios are constructed using 5 highly liquid ETFs each corresponding to a distinct asset class. They are

Figure 4: **Excess Performance of Portfolios in Asset Allocation Setting**

This figure shows the outperformance of yearly-rebalanced KS (solid line, round marker) and AD (dashed line, diamond marker) portfolios in an asset allocation setting. Outperformance is defined as the difference between the portfolio's (KS or AD) return and the corresponding equal-weighted portfolio return. Dates are on the x-axis, outperformance (in %) is on the y-axis.
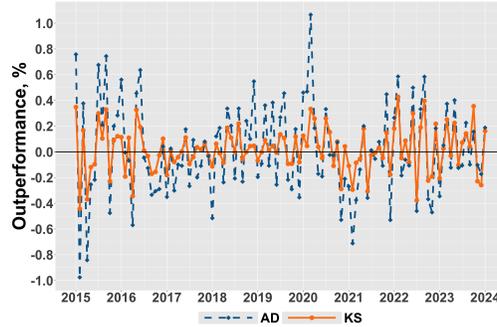
SPY (U.S. Equities), VNQ (U.S. REITs), EFA (Developed Market Equities), TLT (Long Treasuries), GLD (Gold, Liquid Alternatives). The portfolios are rebalanced either monthly (Panel A) or yearly (Panel B). Consistent with the earlier results, the dissimilarity portfolio also outperforms the 1/N rule in an asset allocation setting. With monthly rebalancing, the dissimilarity heuristic delivers excess returns of $6.86 - 6.51 = 35$ bps (KS) and 25 bps (AD). Under annual rebalancing, the corresponding outperformance is 32 bps (KS) and 24 basis points (AD). This excess performance is achieved while keeping Sharpe ratio, tail risk, and turnover comparable to the 1/N benchmark. Figure 4 also shows that this outperformance is not concentrated in any specific period or driven by a single event. However, both the KS and AD portfolios appear to benefit slightly from the COVID-19 pandemic, consistent with the earlier observation that the dissimilarity heuristic tends to have the largest impact during major market events. Finally, Table 9 presents the weights of the KS (Panel A) and AD (Panel B) portfolios. The dissimilarity heuristic produces diversified allocations, making it a viable substitute for the 1/N rule in risk-seeking applications. Allocations to international developed markets (EFA) remain consistently below 0.2 and are lower for the AD portfolios than for the KS portfolios, highlighting that the benefits of international diversification are sensitive to the tails of the return distribution (You and Daigler (2010)).

Table 9: **Asset Allocation Weights**

This table shows the asset allocation weights for the yearly-rebalanced KS (Panel A) and AD (Panel B) in an asset allocation setting. The evaluation period spans 2015-2024, with the table reporting weights at the beginning of each corresponding year.

	SPY	EFA	TLT	VNQ	GLD
		Panel A: KS Statistic			
2015	0.195	0.166	0.238	0.218	0.183
2016	0.196	0.172	0.239	0.227	0.166
2017	0.215	0.182	0.225	0.209	0.169
2018	0.229	0.165	0.226	0.194	0.187
2019	0.240	0.157	0.216	0.192	0.195
2020	0.247	0.163	0.198	0.177	0.215
2021	0.237	0.166	0.189	0.166	0.243
2022	0.225	0.161	0.200	0.171	0.243
2023	0.235	0.166	0.191	0.180	0.229
2024	0.239	0.171	0.197	0.181	0.213
		Panel B: AD Statistic			
2015	0.190	0.122	0.280	0.245	0.163
2016	0.190	0.124	0.285	0.244	0.157
2017	0.203	0.136	0.271	0.227	0.164
2018	0.219	0.126	0.269	0.197	0.188
2019	0.257	0.113	0.258	0.198	0.174
2020	0.280	0.117	0.232	0.165	0.207
2021	0.267	0.120	0.212	0.134	0.267
2022	0.265	0.118	0.221	0.143	0.253
2023	0.273	0.116	0.234	0.162	0.215
2024	0.267	0.134	0.218	0.182	0.199
1/N	0.200	0.200	0.200	0.200	0.200

4.2   Individual Equity Portfolios

The final question is whether the dissimilarity heuristic is effective for portfolios constructed from individual equity returns. To explore this, I form dissimilarity portfolios from the Dow 30 constituents as of November 8, 2024, and evaluate their performance over 2019-2024. The results, reported in Table 10, show that both the KS and AD portfolios substantially outperform the 1/N rule, with excess returns reaching $24.20 - 18.41 = 5.79\%$. Much of this outperformance is driven by NVIDIA, whose rapid price appreciation is captured by

the dissimilarity heuristic. As NVIDIA's return distribution diverges from the rest of the portfolio, the heuristic assigns it progressively higher weights. This scenario illustrates an ideal case for the dissimilarity approach: it performs particularly well when the portfolio is constructed from a carefully curated set of assets with limited downside risk (e.g., the Dow, consisting of large, stable, and generally profitable companies) and one or a few assets subsequently experience substantial growth. From a practitioner's standpoint, restricting the asset universe offers scope for product differentiation and performance enhancement through asset selection. More generally, by focusing on a broad and diversified universe of relatively low-risk assets, the methodology naturally amplifies the impact of best-performing constituents. In other words, broad and diversified universe ensures that the performance of a portfolio approaches that of a market, but when a constituent starts to rapidly appreciate in value, the dissimilarity heuristic dynamically increases its weight. Given its low turnover, this behavior makes the heuristic attractive for both active portfolio management and the construction of specialized indexes.

Table 10: **Portfolio Performance and Turnover: Individual Stocks**

This table shows the out-of-sample portfolio performance and turnover statistics for the maximum pairwise dissimilarity portfolios constructed using individual equity returns (Dow 30 constituents as of Nov. 8, 2024). The portfolios are rebalanced either monthly (Panel A) or yearly (Panel B) and the evaluation period spans 2019-2024. The estimation period is set to 10 years (120 monthly returns). The mean, standard deviation, geometric mean (GeoM), and Sharpe ratio (SR) are annualized. Minimum, mean, and maximum portfolio turnover statistics are reported on a per-reallocation basis, account for the market drift, and reflect the actual trading required to implement a new set of weights. VaR and CVaR stand for 95% Value-at-Risk, and Conditional Value-at-Risk respectively. All statistics except for the Sharpe ratio are expressed as percentages (i.e., 18.41 stands for 18.41%).

	Mean	SD	GeoM	SR	VaR (95%)	CVaR (95%)	Turnover Min	Turnover Mean	Turnover Max
Panel A: Monthly Rebalancing									
KS	19.33	17.26	19.41	0.99	−6.85	−9.01	1.32	2.42	4.68
AD	21.64	17.64	22.07	1.10	−6.99	−9.06	1.44	2.63	5.23
1/N	17.71	17.11	17.55	0.90	−6.79	−8.96	1.30	2.30	4.72
Panel B: Yearly Rebalancing									
KS	20.49	17.18	20.80	1.06	−6.74	−8.90	5.58	9.94	13.15
AD	23.40	17.71	24.20	1.19	−6.89	−8.91	6.70	11.67	15.97
1/N	18.43	16.96	18.41	0.95	−6.68	−8.83	5.65	9.19	11.28

5 Conclusion

In this paper, I introduce a simple long-only portfolio construction rule that weights each asset in proportion to its empirical dissimilarity. To measure dissimilarity, I use the nonparametric Kolmogorov-Smirnov (KS) and Anderson-Darling (AD) statistics, which are generally robust to measurement error, capture the entire return distribution rather than a limited number of moments, and do not explicitly or implicitly rely on any assumed return-generating process or factor structure. Under a sector rotation strategy, the dissimilarity portfolios outperform the 1/N benchmark while maintaining comparable diversification, volatility, tail risk, and turnover. I further validate these findings in both asset allocation and individual equity portfolio settings. Across these settings, the dissimilarity heuristic delivers out-of-sample average annual excess performance ranging from 6 to 579 basis points. Dissimilarity portfolios perform well when constructed from a curated set of assets with limited downside risk. When one or a small number of assets experience price appreciation, their dissimilarity relative to the rest of the universe increases, leading to higher portfolio weights. By limiting downside risk, substantial price appreciation becomes more likely than large depreciation, contributing to long-run performance improvement relative to the 1/N benchmark.

There are other applications beyond the scope of this paper where the proposed portfolio construction mechanism can be of tangible benefit. First, it can be used as a performance attribution tool. Growth-oriented portfolio managers often cite market outperformance as a key objective and deliberately increase overall portfolio risk in order to meet it. However, due to the low-volatility phenomenon, such strategies may still show negative alpha relative to equal-weighted or capitalization-weighted benchmarks. Holding the asset universe and rebalancing schedule fixed, a comparison between an actively managed portfolio and a dissimilarity-based strategy would isolate the performance of active bets relative to a rules-based approach that is also explicitly designed to seek higher risk.

Additionally, the dissimilarity-based allocation can serve as an input during the portfolio construction stage. In a July 14, 2022 webinar, Dan diBartolomeo, president of Northfield Services (a widely used portfolio construction toolkit provider), noted that the solution

to finding an optimal portfolio is a blend of the Markowitz weights and equal weights. A dissimilarity-based portfolio offers a viable alternative to the equal weighting. It retains all desirable properties (such as diversification and robustness to estimation error) and, due to the nonparametric nature of dissimilarity functions, directly incorporates asset characteristics outside of the asset means and variances.

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